

Examples for the BFGS Quasi-Newton Update

$$\text{Minimize } f(x) = e^{x_1-1} + e^{-x_2+1} + (x_1 - x_2)^2$$

Iteration 1:

$$x^0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (\text{initial point})$$

$$B^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$g^0 = \begin{pmatrix} 0.3679 \\ -2.7183 \end{pmatrix}$$

s^0 is the solution of $B^0 s^0 = -g^0$

$$s^0 = -B_0^{-1} g^0 = \begin{pmatrix} -0.3679 \\ 2.7183 \end{pmatrix}$$

$$x^1 = x^0 + \alpha_0 s^0,$$

Line search with Wolf Condition gives

$$\alpha_0 = 1$$

$$x^1 = \begin{pmatrix} -0.3679 \\ 2.7183 \end{pmatrix}$$

$$g^1 = \begin{pmatrix} -5.9178 \\ 5.9930 \end{pmatrix}$$

$$\sigma^0 = x^1 - x^0 = \begin{pmatrix} -0.3679 \\ 2.7183 \end{pmatrix}$$

$$y^0 = g^1 - g^0 = \begin{pmatrix} -6.2856 \\ 8.7113 \end{pmatrix}$$

$$\Delta B^0 = \frac{y^0(y^0)^T}{(\sigma^0)^T y^0} - \frac{B^0 \sigma^0 (\sigma^0)^T B^0}{(\sigma^0)^T B^0 \sigma^0} = \begin{pmatrix} 1.5020 & -1.9737 \\ -1.9737 & 1.9376 \end{pmatrix}$$

$$B^1 = B^0 + \Delta B^0 = \begin{pmatrix} 2.5020 & -1.9737 \\ -1.9737 & 2.9376 \end{pmatrix}$$

Iteration 2:

s^1 is the solution of $B^1 s^1 = -g^1$

$$s^1 = -B_1^{-1} g^1 = \begin{pmatrix} 1.6082 \\ -0.9596 \end{pmatrix}$$

$$x^2 = x^1 + \alpha_1 s^1,$$

Line search with Wolf Condition gives

$$\alpha_1 = 1.25$$

$$x^2 = \begin{pmatrix} 1.6423 \\ 1.5188 \end{pmatrix}$$

$$g^2 = \begin{pmatrix} 2.1478 \\ -0.8422 \end{pmatrix}$$

$$\sigma^1 = x^2 - x^1 = \begin{pmatrix} 2.0102 \\ -1.1995 \end{pmatrix}$$

$$y^1 = g^2 - g^1 = \begin{pmatrix} 8.0656 \\ -6.8353 \end{pmatrix}$$

$$\Delta B^1 = \frac{y^1(y^1)^T}{(\sigma^1)^T y^1} - \frac{B^1 \sigma^1 (\sigma^1)^T B^1}{(\sigma^1)^T B^1 \sigma^1} = \begin{pmatrix} 0.3711 & 0.0646 \\ 0.0646 & -0.4386 \end{pmatrix}$$

$$B^2 = B^1 + \Delta B^1 = \begin{pmatrix} 2.8731 & -1.9091 \\ -1.9091 & 2.4990 \end{pmatrix}$$

Iteration 3:

s^2 is the solution of $B^2 s^2 = -g^2$

$$s^2 = -B_2^{-1} g^2 = \begin{pmatrix} -1.0635 \\ -0.4754 \end{pmatrix}$$

$$x^3 = x^2 + \alpha_2 s^2,$$

Line search with Wolf Condition gives

$$\alpha_2 = 1.0313$$

$$x^3 = \begin{pmatrix} 0.5456 \\ 1.0285 \end{pmatrix}$$

$$g^3 = \begin{pmatrix} -0.3310 \\ -0.0061 \end{pmatrix}$$

$$\sigma^2 = x^3 - x^2 = \begin{pmatrix} -1.0967 \\ -0.4903 \end{pmatrix}$$

$$y^2 = g^3 - g^2 = \begin{pmatrix} -2.4788 \\ 0.8361 \end{pmatrix}$$

$$\Delta B^2 = \frac{y^2 (y^2)^T}{\sigma_2^T y^2} - \frac{B^2 \sigma_2 \sigma_2^T B^2}{\sigma_2^T B^2 \sigma_2} = \begin{pmatrix} 0.2127 & 0.0625 \\ 0.0625 & -0.0737 \end{pmatrix}$$

$$B^3 = B^2 + \Delta B^2 = \begin{pmatrix} 3.0858 & -1.8467 \\ -1.8467 & 2.4253 \end{pmatrix}$$

Iteration $k...$